Outline
6. Density
7. Sea ice, freezing point
8. Potential density
9. Isopycnal analysis & Neutral density
10. Tracers on isopycnal surfaces
11. Stability, Brunt-Vaisala frequency
12. Sound speed
6. Seawater density ($\rho$)

Density = Mass/Volume

$\text{kg/m}^3 = \text{weight (kg) per space (m}^3\text{)}$

Seawater density ($\rho$) depends on Salinity ($S$), Temperature ($T$), and Pressure ($p$)

$$\rho = \rho(S, T, p) \quad \text{units are mass/volume (kg/m}^3\text{)}$$

$$\sigma = \rho - 1000 \text{ kg/m}^3 \quad \text{convenient quantity, to avoid writing ‘1000’ in densities}$$

Side note: Specific volume (alpha) $\alpha = 1/\rho$ (units are volume/mass (m$^3$/kg))

Equation of state (EOS)

Seawater density is determined empirically with lab measurements.

New references: TEOS-10 and Millero et al. (linked to notes)

Old EOS UNESCO tables (computer code in fortran, matlab, c) (see link to online lecture notes)
6. Equation of state (EOS) for seawater

- Common way to express density is as an anomaly ("sigma")
  \[ \sigma (S, T, p) = \rho(S, T, p) - 1000 \text{ kg/m}^3 \]

- The EOS is **nonlinear**

- This means it contains products of T, S, and p with themselves and with each other (i.e. terms like \( T^2, T^3, T^4, S^2, TS \), etc.)
6. Seawater density $\rho$

Seawater density increases with

1) colder $T$, higher $S$,

2) Higher pressure

3) Pure water has a maximum density at $4^\circ$ C, atmospheric pressure of

$$\rho(0,4^\circ \text{C,1bar}) = 1000 \text{ kg/m}^3 = 1 \text{ g/cm}^3$$

4) Effect of salinity on freezing point $T_f$
6. Surface density ($\rho - 1000 \text{kg/m}^3$) (winter)

Seawater density $\rho$ at the sea surface ranges from about
1021 kg/m$^3$ (tropics, warmest)
to
1028 kg/m$^3$ (highest latitudes, coldest)

Surface density $\sigma_b$ (kg m$^{-3}$) in winter (January, February, and March north of the equator; July, August, and September south of the equator) based on averaged (climatological) data from Levitus and Boyer (1994) and Levitus et al. (1994b).

DPO Figure 4.16
6) Seawater density vertical profile

North Pacific profiles
Temperature, potential temperature, salinity

Density \( \sigma = \rho - 1000 \) (kg/m\(^3\))
With pressure effect removed: ‘potential density’

North Pacific profile
36° 30’N, 135°W

FIGURE 3.3

FIGURE 3.6
Seawater density on a Temperature-Salinity diagram

- Temperature and density of ocean water are inversely related: warm water means low density, cold water means high density.
- Density and salinity are directly related: the larger the salinity, the denser the water.
- The effect of salinity on density is a function of temperature - temperature variation dominate changes in density in warm water and salinity variations dominate in cold water.
Seawater density (at sea surface, p=1 atmosphere)

Contours of equal density are called: **isopycnals**. (iso = constant, pycnal = density)

- Mean temperature and salinity are 3.5°C and 34.6 psu
- Is salinity or temperature more important in the ocean?
Volume of the ocean in temperature/salinity space

*Note contours are in sigma: \( \sigma = \rho - 1000 \) (kg/m\(^3\))

75% of ocean is 0-6°C, 34-35 psu

50% is 1.3-3.8°C, 34.6-34.7 psu (\(\sigma_{\theta} = 27.6\) to 27.7 kg/m\(^3\))

Mean temperature and salinity are 3.5°C and 34.6 psu
6. Equation of state for seawater: definitions

- \( \rho(S, T, p) \)
- Changes in \( \rho \) as a function of \( T, S, p \):

\[
\frac{d\rho}{d\bar{\rho}} = \frac{\partial \rho}{\partial T} dT + \frac{\partial \rho}{\partial S} dS + \frac{\partial \rho}{\partial \rho} d\bar{\rho}
\]

\[
= -\alpha \rho dT + \beta \rho dS + \kappa \rho d\rho
\]

- Thermal expansion coefficient
  
  Generally positive for seawater

- Haline contraction coefficient
  
  Positive

- Adiabatic compressibility
  
  Positive
Seawater density

EOS is non-linear!

Mix water at T=5°C and S = 32 with water T=37°C and S=38, what do I get?

T= (5+27)/2=18.5°C and S=(32+38)/2=35. New density?

*Note contours are in sigma: \( \sigma = \rho - 1000 \) (kg/m\(^3\))
7. Freezing point and sea ice

Most seawater has maximum density at the freezing point. That means it ‘overturns’ (conveects) down to a depth where water below has higher density.

Why then does sea ice float?

Because it is actually less salty and thus less dense than the seawater...

- **Brine rejection**: as sea ice forms, it excludes salt from the ice crystal lattice.
- The salt drips out the bottom, and the sea ice is much fresher (usually ~3-4 psu) than the seawater (around 30-32 psu)
- The rejected brine mixes into the seawater below. If there is enough of it mixing into a thin enough layer, it can measurably increase the salinity of the seawater, and hence its density.
- This is the principle mechanism for forming the densest waters of the world ocean.
- The much fresher sea ice melts the next spring, and makes the surface water much fresher (because the salt mixed down into the waters below).
7. More on sea ice ... Brinicles

Also watch... http://www.youtube.com/watch?v=CSIHYlbVh1c
7. Sea ice formation: polynyas

(a) Wind-forced (latent heat) polynya

Wind

Heat loss

Polynya

Sea ice

Cold, fresher

Dense shelf water

Cold, saltier

Brine rejection

(b) Melting (sensible heat) polynya, driven by mixing

Heat loss

Polynya

Sea ice

Cold, fresher

Tidal mixing

Melting

Vertical excursion

Warm, saltier
Adiabatic compression has 2 effects on density:

(1) Changes temperature (increases it)

(2) Mechanically compresses so that molecules are closer together

As with temperature, we are not interested in this purely compressional effect on density. We wish to trace water as it moves into the ocean. Assuming its movement is adiabatic (no sources of density, no mixing), then it follows surfaces that we should be able to define. This is actually very subtle because density depends on both temperature and salinity.
8. Potential density: compensating for compressibility

**Potential density**: reference the density $\sigma(S, T, p)$ to a specific pressure, such as at the sea surface, or at 1000 dbar, or 4000 dbar, etc.

Steps:
(1) compute the potential temperature **AT THE CHOSEN REFERENCE PRESSURE**
(2) compute density using that potential temperature and the observed salinity at that reference pressure.

That is: for **potential density referenced to the sea surface** ($p=0$):

$$\sigma_\theta = \sigma_0 = \sigma(S, \theta, 0)$$
8. Potential density profiles ($\sigma_0$ & $\sigma_4$): note different absolute range of values because of different reference pressure

Potential density referenced to other pressures:

$\sigma_1$ is potential density referenced to 1000 dbar, calculated using potential temperature $\theta_1$ referenced to 1000 dbar, etc.

$$
\begin{align*}
\sigma_0 &= \sigma_0 = \sigma(S, \theta, 0) \\
\sigma_1 &= \sigma(S, \theta_1, 1000) \\
\vdots \\
\sigma_4 &= \sigma(S, \theta_4, 4000)
\end{align*}
$$

DPO Figure 4.17
8. Atlantic section of potential density $\sigma_\theta$

**Potential** density $\sigma_\theta$ ranges from
1022 kg/m$^3$ at the sea surface
to
1028 kg/m$^3$ at bottom of ocean

**In situ** density $\sigma$ ranges from about
1022 kg/m$^3$ at the sea surface
to
1050 kg/m$^3$ at bottom of ocean, mainly due to compression.
An important nonlinearity for the Equation of State

- Cold water is more compressible than warm water.
- Seawater density depends on both temperature and salinity. (Compressibility also depends, much more weakly, on salinity.)
- Seawaters of the same density $\sigma$ but different $(T, S)$ will compress differently as they move to higher pressure.
- Cold, fresh water will compress more than warm, salty water and therefore if both parcels of seawater are moved to the same pressure, the cold/fresh parcel will end up denser than the warm/salty parcel even though they started out at exactly the same density at their original pressure.
- Constant density surfaces flatten in temperature/salinity space when the pressure is increased (next slide).
8. Potential density: relative to 0 dbar and 4000 dbar

Example:
(1) 2 water parcels with different temperature and salinity and the same density $\sigma_\theta(S, \theta, 0)$ at pressure $p = 0$

At higher pressure, here $p = 4000$ dbar, the 2 parcels have compressed differently.

The colder parcel compresses more. So at 4000 dbar, its density $\sigma_4(S, \theta_4, 4000)$ is greater than the density of the warmer parcel. This is an adiabatic process (no addition or removal of heat or salinity)
8. Potential density: relative to 0 dbar and 4000 dbar

Example:
(1) 2 water parcels with different temperature and salinity and the same density $\sigma_0(S, \theta, 0)$ at pressure $p = 0$

At higher pressure, here $p = 4000$ dbar, the 2 parcels have compressed differently.

The colder parcel compresses more. So at 4000 dbar, its density $\sigma_4(S, \theta_4, 4000)$ is greater than the density of the warmer parcel. This is an adiabatic process (no addition or removal of heat or salinity)

(2) 2 water parcels with different temperature and salinity with slightly different densities at pressure $p = 0$. Cold/fresh parcel is less dense than warm/salty at $p=0$. But at $p=4000$, the cold/fresh parcel is denser. This effect is called thermobaricity.
8. Atlantic section of potential density $\sigma_0$

Note apparent instability of this permanent density distribution! Looks like dense water above lighter water. The ocean does not suffer instability like this for more than a few hours – it mixes (convects). Cannot be permanently unstable.

-> use deeper reference pressures to check local vertical stability (e.g. $\sigma_4$)
8. Atlantic section of potential density referenced to 4000 dbar (sea surface): $\sigma_4$

Potential density $\sigma_\theta$ inversion vanishes with use of deeper reference ($\sigma_4$): in fact, extremely stable (many contours)!!
9. Isopycnal analysis: track water parcels through the ocean

- Parcels move mostly adiabatically (isentropically). Mixing with parcels of the same density is much easier than with parcels of different density, because of ocean stratification.

- Use isopycnal surfaces as an approximation to isentropic surfaces.

- Because of thermobaricity (cold water more compressible), none of the potential density surfaces is an isentropic surface – we cannot track water from the surface to the deep ocean along them. Must continually change our reference pressure for comparison.
9. Neutral density $\gamma^n$

- To follow a water parcel as it travels down and up through the ocean:
- Must change reference pressure as it changes its depth, in practical terms at least every 1000 dbar

- Neutral density provides a continuous representation of this changing reference pressure. (Jackett and McDougall, 1997)
- Ideal neutral density: follow actual water parcel as it moves, and also mixed (change T and S). Determine at every step along its path where it should fall vertically relative to the rest of the water. This is the true path of the parcel.
- Practically speaking we can’t track water parcels because they actually do mix.
9. Atlantic section of “neutral density”: $\gamma^n$

"Neutral density" surfaces are defined to be closest to isentropic surfaces that parcels would follow.

Note that we retain vertical stability when using neutral density.
10. Isopycnal analysis: isoneutral surface in the Pacific Ocean

**Depth**

Depth (m) $27.6 \gamma^n$ (kg/m³)

**Salinity**

Salinity (PSS78) $27.6 \gamma^n$ (kg/m³)

WHP Pacific Atlas (Talley, 2007)
10. Isopycnal analysis: isoneutral surface in the Pacific Ocean

Potential temp.

Potential temperature (°C) 27.6 $\gamma^n$ (kg/m$^3$)

Salinity

Salinity (PSS78) 27.6 $\gamma^n$ (kg/m$^3$)

WHP Pacific Atlas (Talley, 2007)
10. Tracers

• Use tracers to help determine pathways of circulation, age of waters

• Conservative vs. non-conservative
  • Conservative tracers do not interact with their environment except by mixing, and at their localized sources. Examples are salinity, potential temperature, potential density, chlorofluorocarbons
  • Non-conservative tracers are changed chemically or biologically within the water column. Many examples: oxygen, nutrients

• Radioactive vs. stable

• Natural vs. anthropogenic

• We will return to this topic in “Typical distributions” lectures
10. Tracers on isoneutral surfaces

Oxygen

\[
\text{Oxygen (\mu mol/kg) } 27.6 \gamma^n (\text{kg/m}^3)
\]

Chlorofluorocarbons

\[
\text{CFC-11 (pmol/kg) } 27.6 \gamma^n
\]

http://woceatlas.ucsd.edu

WHP Pacific Atlas (Talley, 2007)
10. Tracers on isoneutral surfaces

$\delta^3\text{He}$

$\delta^3\text{He} \ (\%) \ 27.6 \gamma \ ^n \ (\text{kg/m}^3)$

$\Delta^{14}\text{C}$

$\Delta^{14}\text{C} \ (/\text{mille}) \ 27.6 \gamma \ ^n \ (\text{kg/m}^3)$

http://woceatlas.ucsd.edu

WHP Pacific Atlas (Talley, 2007)
11. Water Column stability

Lighter water sits above denser water: the ocean is vertically stratified.

(There can be episodic mixing events when it is unstably stratified for a few hours, but it will convect and become stable or neutrally stable.)

Internal waves: consequence of stratification. Waves ‘ride’ the internal stratification.
12. Internal wave or Brunt-Vaisala frequency

- Internal waves are (mostly) gravity waves
- Internal waves have a frequency (period is time between successive crests, frequency is 1/period or $2\pi$/period)
11. Internal wave or Brunt-Vaisala frequency

- **Vertical stratification**: density $\rho$ increases with depth $z$.
- Stratification is quantified by the measured value of $\Delta \rho / \Delta z$
- Gravity $g$ is the restoring force on the water.
- If water is displaced vertically, it oscillates in an internal wave with frequency that depends on $g$ and on the stratification:

$$N = \sqrt{-\frac{g}{\rho} \times \Delta \rho / \Delta z}$$

- $N$ is called the **Brunt-Vaisala frequency**. Units are 1/time = 1/sec.
- If the water is more stratified, $N$ is higher. If less stratified, frequency is lower.
FIGURE 3.6

(a) Potential density and (b) Brunt-Väisälä frequency (cycles/h) and period (minutes) for a profile in the western North Pacific.
11. Brunt-Vaisala frequency

Values of Brunt-Vaisala frequency:
0.2 to 6 cycles per hour

These are the frequencies of “internal waves”

Compare with frequency of surface waves, which is around 50-500 cycles per hour (1 per minute to 1 per second)

Internal waves are much slower than surface waves since the internal water interface is much less stratified than the sea-air interface, which provide the restoring force for the waves.
Brunt Vaisala frequency distribution

Potential density $\sigma_2$ relative to 2000 dbar

Brunt Vaisala frequency (cycles per hour)

Talley SIO210 (2019)
12. Sound in water

- Sound is a compressional wave
- Sound speed is much higher in water than in air because water is much less compressible than air
12. Sound speed profile: contributions of temperature and pressure to variation of \( c_s \)

Adiabatic compressibility of seawater \[ \kappa = \rho^{-1} (\partial \rho / \partial p)_{\theta,s} \]

(if compressibility is large, then \( c_s \) is small; if compressibility is small then \( c \) is large)

Sound speed: \[ C_s = (\kappa \rho)^{-1/2} \]

- Warm water is less compressible than cold water, so sound speed is higher in warm water
- Water at high pressure is less compressible than water at low pressure, so sound speed is higher at high pressure
- These competing effects create a maximum sound speed at the sea surface (warm) and a maximum sound speed at great pressure, with a minimum sound speed in between
- The sound speed minimum is an acoustic waveguide, called the “SOFAR” channel
12. Sound channel, or SOFAR channel (a wave guide)