SIO 210 Problem Set 1 Answer Key with rubric (Oct. 14, 2019)
October 3, 2019
Due October 10, 2019 (1 week)

If you work together on these, please make sure that you understand the concepts and use your group discussion to help with the understanding.

1. 25 points
(a) 5 points Explain briefly the usefulness of non-dimensional parameters.
(general answer): Non-dimensional parameters allow us to compare phenomena, which in the ocean have very large ranges of values. For instance, from Figure 1.2, we can see that length scales range from small bubbles to the global scale, and time scales from milliseconds to 100,000s of years. From descriptions in class, vertical length scales range from bubbles to the depth of the ocean. When we look at the physical processes for a given phenomenon, for instance for a bubble, or for tides, or for the wind-driven circulation, it is most useful to first look at its non-dimensional parameters which tell us whether, for instance: is its time scale long or short relative to the Earth's rotation time scale? (If longer than the rotation time scale, then we have to include rotation when we describe the physics.) etc.

For the following, using information from the introductory lecture slides and Chapter 1, give a typical height scale H, horizontal length scale L, and time scale T. Estimate two non-dimensional parameters: '*aspect ratio*' and '*Rossby number*'. For each one, briefly describe what you expect the ratio of vertical velocity to horizontal velocity to be.

Second (a) 5 points Gulf Stream (use width across the Gulf Stream for L rather than an along-stream lengthscale; also the Gulf Stream reaches to the bottom)
H = 5 km. L = 100 km. T = many years, can choose from 1 year (seasonal time scales) to 1000 years or longer.

(b) 5 points El Niño/La Niña
(c) 5 points Global overturning circulation
(d) 5 points Internal wave

Can answer this question with a table. Note that you can choose length and time scales within a range that suits the phenomenon. What matters is the size of the non-dimensional parameter relative to unity (1).

	H	L	T	Aspect ratio = H/L	Rossby number = T_{rot}/T
(a) Gulf Stream	5 km	100 km	1 year to 1000 years	5km/100km = 0.05 << 1	1day/365 days = $2x10^{-3}$ << 1 1 day/(1000*365 days) = $2x10^{-6}$ << 1 etc

El Nino/La Nina	1 km	10,000 km	5 years	1km/10,000km = $10^{-4} << 1$	1 day/(5*365 days) = $5 \times 10^{-4} << 1$
Global overturning circulation	5 km	10,000 km	1000 years	1km/10,000km = $10^{-4} << 1$	1 day/(1000*365 days) = $2 \times 10^{-6} << 1$
Internal wave	10 m	10 m	2 hours	10m/10m = 1	1 day/2 hours = 12 >> 1

The ratio of vertical to horizontal velocity is the same order of magnitude as the aspect ratio, in each case.

2. 19 points + 5 extra
This potential temperature/salinity diagram shows contours of potential density. The large dot at lower temperature and salinity (cold, fresh) is the property of water flowing into the North Atlantic from the Nordic Seas, over the sill (deepest point) in Denmark Strait between Greenland and Iceland. The warm, salty dot is the water flowing into the North Atlantic from the Mediterranean, over the sill in the Strait of Gibraltar between Africa and Europe. (Although this does not matter for the problem, it is interesting to know that both sill depths are about the same: 500 m.)

(a) 4 points What is the potential density relative to the sea surface σ_θ of these two water parcels?___1028 kg/m^3_____________
(This was a bad choice of dots on my part: there is very little water in the ocean that has density that is this high.)

(b) 5 points On the diagram, sketch in the contours of potential density σ_4 relative to 4000 dbar (that is, relative to a pressure higher than the sea surface pressure). This does not need to be exact, but please get the relative slope/angle of the contours.
Draw contours that are similar to the sigma theta contours, but rotated to be flatter.

Briefly explain why the σ_4 contours are rotated relative to the σ_θ contours.
Because warm water is less compressible than cold water, when water parcels of the same surface density are submerged to a higher pressure, in this case 4000 dbar, the cold water will be denser than the warm water. This means that when you draw contours of potential density relative to 4000 dbar, the contour that goes through the warmer parcel will have lower sigma_4 than the contour that goes through the colder parcel, which immediately gives you the general picture of flatter sigma_4 contours compared with sigma_0 contours.

(c) 5 points Relative to 4000 dbar, is the potential density σ_4 of the Mediterranean parcel **HIGHER** or **LOWER** than the potential density σ_4 of the Nordic Seas parcel? (circle one)
Mediterranean parcel is LOWER sigma_4

Again explain briefly.

Mediterranean outflow is warmer/saltier than Nordic Seas outflow, and so when they both descend to greater pressure, the Mediterannean parcel compresses less, and is therefore less dense than the Nordic Seas parcel.

(d) 5 points The vertical axis is potential temperature referenced to the sea surface. This is not the same as the measured temperature. Is the *measured* temperature of both parcels

HIGHER (WARMER) or **LOWER (COLDER)**

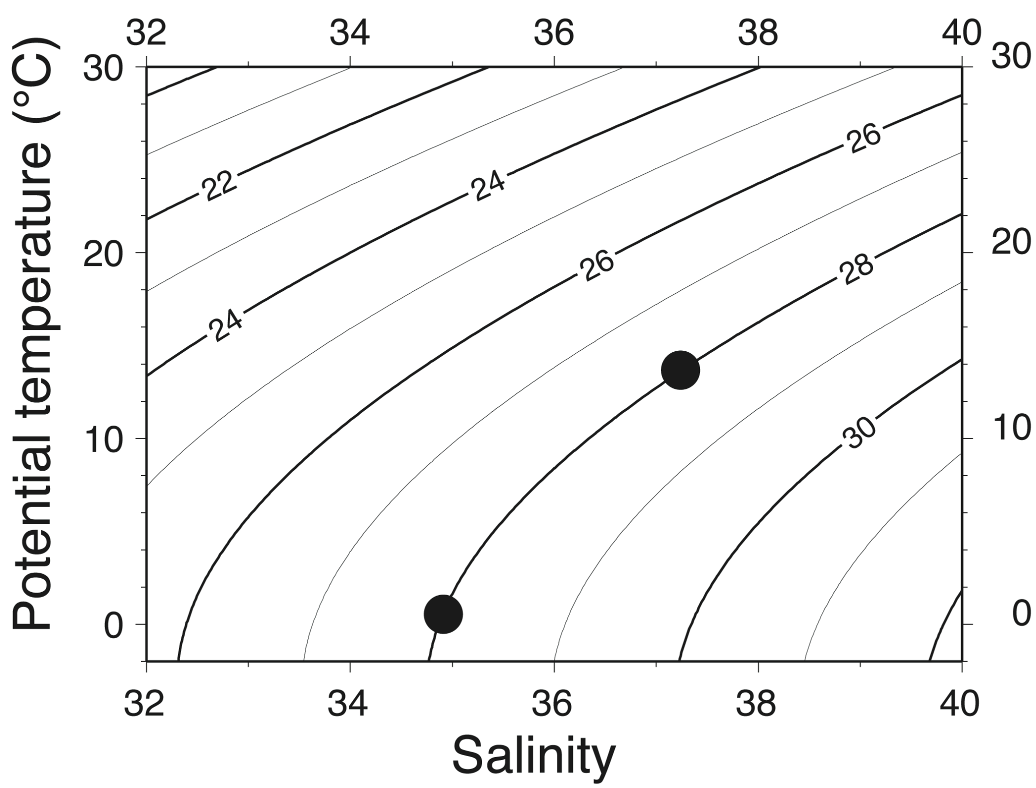
than their potential temperature?

Explain this briefly.

Measured temperature is higher than the potential temperature referenced to the sea surface. When a parcel of water within the water column is moved to the sea surface adiabatically and without change of salinity (which is the definition of potential temperature), it will expand and therefore its temperature will decrease.

e) 5 points Extra: is the potential temperature referenced to 4000 dbar of these parcels lower or higher than the potential temperature shown here (referenced to the sea surface)?

When these parcels are moved DOWN to 4000 dbar, they will be compressed and their temperature will rise. Therefore the potential temperature relative to 4000 dbar will be higher than the potential temperature relative to the sea surface.



(3. skipped)

4. 20 points

a) 5 points What forces balance each other in hydrostatic balance?

__pressure gradient force____________ and ___gravitational force__________ (words)

b) 5 points Write these out in symbolic form (from lecture).

___-dp/dz__________ and ___-ρ g__________ (terms in equation)

(Ray: I can't figure out how to write partial derivatives in the first term using fonts, but students should write this as a partial dp/dz. They can also write these in the form –(1/rho)dp/dz and -g, or write them on opposite sides of an equation, so with different signs.)

c) 10 points The density of seawater is approximately 1025 kg/m^3. Using hydrostatic balance and this density, find the pressure at a depth of 4000 m. (Use gravity g = 9.8 m/sec^2. Ignore compressibility of seawater for this question.)

delta p = (delta z) ρ g = (4000 m)(1025 kg/m^3)(9.8 m/sec^2) = 4.018 x 10^7 kg/m sec^2 = 4.018 x 10^7 N/m^2 = 4.018 x 10^3 dbar

5. 26 points A potential density section from the Atlantic is shown. (from http://whp-atlas.ucsd.edu/atlantic/a16/sections/printatlas/printatlas.htm)

For the top 0-1000 m, the contoured property is potential density relative to the sea surface (0 dbar).

(a) 4 points What is the lowest potential density at the sea surface and where is it found (latitude)? (It will be easier to see if you go to the online section. You will find postage stamp plots of all of the properties – click on the potential density plot.)

Lowest density at the sea surface is less than 23.00 kg/m^3, in two locations: between 5S and equator, and between 3N and 7N.

(b) 4 points Using the vertical section, and the location of lowest potential density, what is the potential density at 150 m depth?

Looking at 5S to the equator: potential density is 26.6 or 26.7 kg/m^3

At 3N-7N: 26.7 kg/m3

(c) 7 points Estimate the Brunt-Vaisala frequency N for this location and depth range, using values from (a) and (b).

N = sqrt(-(g/rho) drho/dz)

Make sure to use complete rho in the denominator (including the 1000 kg/m3)

rho = mean rho = (1023.0 + 1026.6)/2 = 1024.8 kg/m^3

drho = 3.6 kg/m^3

dz = 150 m

N = sqrt (-[(9.8 m/sec^2)/(1024.8 kg/m3)] * (3.6 kg/m^3)/150 m) = 0.015/sec = 54 cycle/hr.

This is quite high, but is the right estimate.

(d) 7 points Do the same three steps for the density/N at 25°S.

Surface density is about 24.5

150 m density is about 25.5
Rho = 25.0
Drho = 1.0
N = sqrt (-[(9.8 m/sec^2)/(1025.0 kg/m3)] * (1.0 kg/m^3)/150 m) = 0.008/sec = 29 cycle/hr

e) 4 points Compare the results from (c) and (d). Which has higher frequency? Explain.
Higher frequency is from the low latitude location because the stratification is much higher.

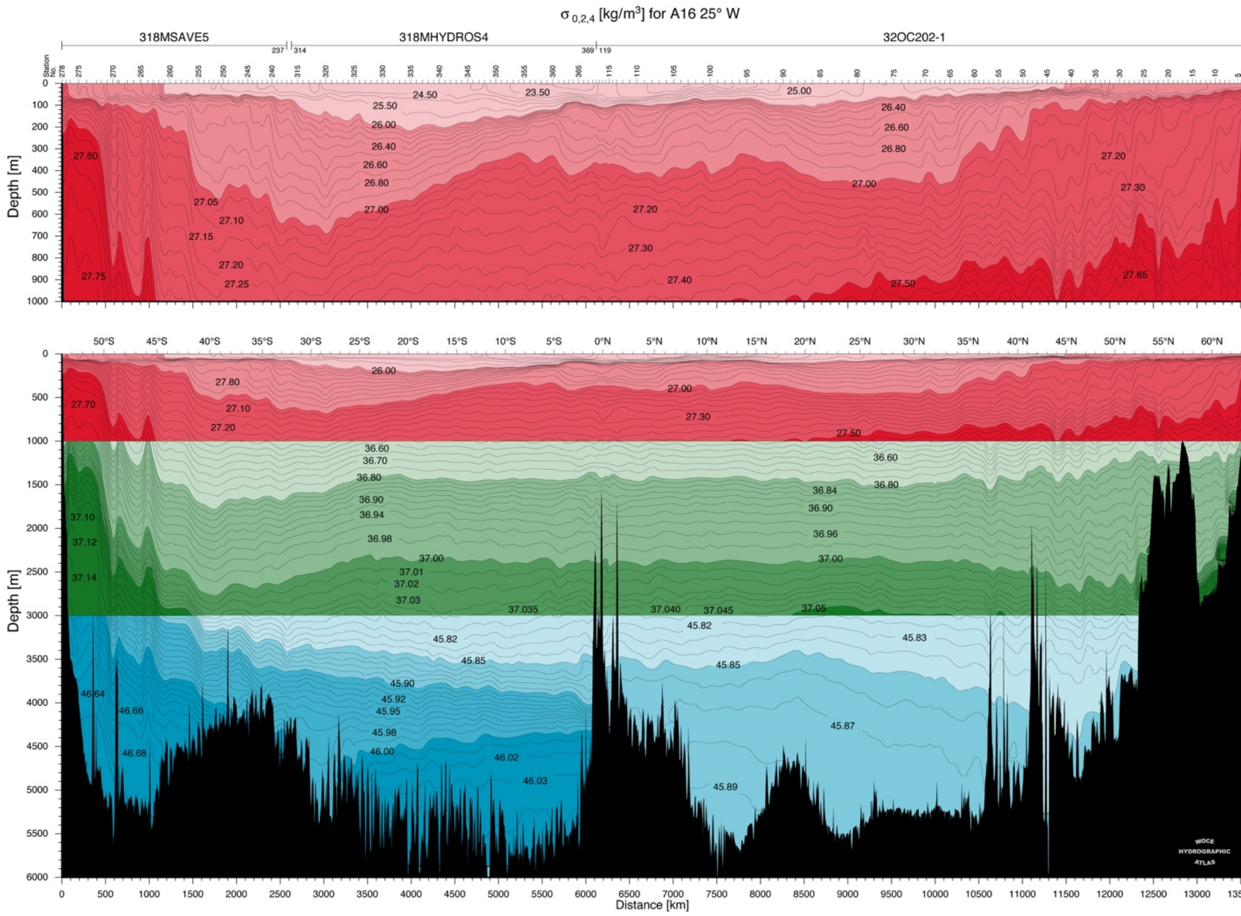
6. 10 points
Using the same potential density section:
Note that in the lower panel there are three differently colored portions. The top is potential density relative to the sea surface (p=0) σ, the middle is relative to 2000 dbar (σ_2) and the bottom is relative to 4000 dbar (σ_4).

(a) 5 points Compare the ranges of densities in the three panels. Why is the range so different?
Range in upper (red) panel: potential densities are 23 to 27.8
Range in middle (green) panel: potential densities are 36.5 to 37.14
Range in lower (blue) panel: potential densities are 45.8 to 46.7

Densities are in the mid-20s, 36's, and 45's. The overall difference between these is due to compressibility due to pressure – density is mechanically higher because of pressure.

The overall range in each panel is much larger for the surface (red) panel because this is the upper ocean where stratification is much higher than in the abyssal ocean, so the density range is larger. The ranges in the green and blue panels are similar: both are abyssal and reflect the lower vertical stratification of the deep ocean.

(b) 5 points Where are the highest σ_4 values found? Why is density so high here? What is the source of the dense water?
Compare this high density with highest density in the other hemisphere (your first answer will be from either the southern or northern hemisphere - compare here with the other hemisphere). Why is there a difference between the two hemispheres?
Highest values are around 47.0 kg/m^3 at the bottom of the southern end of the section.
I just realized that there is a bad label on a contour there – the 46.64 label should be 47.0, just looking at the contours.
This is the densest water, reflecting production of the densest waters in the Atlantic along the coast of Antarctica. From one of the PPSW lectures, this is due to the waters being at the freezing point, and subject to large brine rejection. The water mass name is Antarctic Bottom Water, which we will be introducing in Thursday's lecture.


σ 0,2,4 [kg/m3] for A16 25° W
318MSAVE5
318MHYDROS4
32OC202-1
Depth [m]
Distance [km]
WOCE HYDROGRAPHIC ATLAS