Closed book. (100 total points). One sheet (both sides) of your own notes is allowed. A simple calculator is allowed. I will provide a limited number. No electronics with communications.

**Possibly useful expressions and values**
- $1 \text{ Sv} = 1 \times 10^6 \text{ m}^3/\text{sec}$
- $1^\circ \text{ latitude} = 111$ km
- $g = 9.8 \text{ m/s}^2$
- $\rho c_p T$
- $\rho = 1025 \text{ kg/m}^3$
- $c_p = 4000 \text{ J/kg}^\circ \text{C}$
- $1 \text{ PW} = 10^{15} \text{ W}$
- $H = \rho c_p V(T_o - T_i)$
- $F \sim \rho V(S_o - S_i)/S_m$

**Multiple choice** (5 problems, 2 points each, 10 points total)
For each problem, **circle the CORRECT answer**. (There should be only one.)

1. For the Gulf Stream (or Kuroshio), the approximate aspect ratio $H/L$, where $H$ is the height of the phenomenon and $L$ is its length scale, is
   (a) 1
   (b) 1000
   (c) 0.05
   (d) 0.00001
   I appreciated after grading that there are 2 right answers, depending on whether students chose the length scale across stream (100 km) or along stream (say 5000 km)

2. Salinity in sea ice compared with open ocean salinity (sea ice salinity/ocean salinity) is approximately
   (a) 3 psu/30 psu
   (b) 40 psu/35 psu
   (c) 25 psu/40 psu
   (d) 0 psu/30 psu

3. Brunt-Vaisala frequency is high (period is short)
   (a) in the surface mixed layer
   (b) compared with surface wave frequency
   (c) in the abyssal ocean
   (d) compared with mesoscale eddy timescales
4. Which of these makes Eulerian observations?
(a) Surface drifter
(b) Moored current meter at the equator
(c) Glider (autonomous underwater vehicle)
(d) Subsurface float in the Argo program

5. Density
(a) Increases with increasing temperature
(b) Increases with decreasing pressure
(c) Increases with increasing salinity
(d) Has units of kg.

Circle the best answer (5 problems, 2 points each, 10 points total)

6. The temperature at the freezing point of water (increases/decreases) with increasing salinity.

7. An anticyclone rotates counterclockwise around a (High/Low) pressure in the Southern Hemisphere.

8. In hydrostatic balance, (Coriolis force/ gravity) balances the pressure gradient force.

9. Eddy viscosity results from the (molecular/turbulent) motion of water.

10. Inertial motion results from a balance between acceleration and (Coriolis/centrifugal) force.

Problems (80 points)

11. (20 points)
The momentum equations in a fluid in a rotating reference frame are

x: acceleration + advection + Coriolis = pressure gradient force + viscous forces

y: acceleration + advection + Coriolis = pressure gradient force + viscous forces

z: acceleration + advection = pressure gradient force + gravity + viscous forces
or, written mathematically:

\[ 
\begin{align*}
\text{x: } & \partial u/\partial t + u \partial u/\partial x + v \partial u/\partial y + w \partial u/\partial z - fv = - (1/\rho) \partial p/\partial x + \partial/\partial x(A_H \partial u/\partial x) + \partial/\partial y(A_H \partial u/\partial y) + \\
& \quad \partial/\partial z(A_\nu \partial u/\partial z) \\
\text{y: } & \partial v/\partial t + u \partial v/\partial x + v \partial v/\partial y + w \partial v/\partial z + fu = - (1/\rho) \partial p/\partial y + \partial/\partial x(A_H \partial v/\partial x) + \partial/\partial y(A_H \partial v/\partial y) + \\
& \quad \partial/\partial z(A_\nu \partial v/\partial z) \\
\text{z: } & \partial w/\partial t + u \partial w/\partial x + v \partial w/\partial y + w \partial w/\partial z = - (1/\rho) \partial p/\partial z - g + \partial/\partial x(A_H \partial w/\partial x) + \partial/\partial y(A_H \partial w/\partial y) + \\
& \quad \partial/\partial z(A_\nu \partial w/\partial z)
\end{align*} \]

(a) Which pairs of terms yield geostrophic balance? (circle and label) Coriolis and pgf in both the x and y momentum equations.

(b) Briefly describe what the Coriolis force is. Coriolis force is a pseudo-force that arises because of the rotating frame of reference we normally use to describe the ocean and atmosphere (in the reference frame fixed to local coordinates on the rotating Earth). In the NH, Coriolis force appears to push objects/water/air to the right of the direction of their motion; in the SH, it appears to push them to the left. This is because the objects would move in a straight line if subjected to no external actual forces (such as pdf), but the rotating reference frame is rotating out from under the object moving in a straight line; therefore its path is curved relative to the rotating frame.

(c) The map shows sea surface height in the Pacific Ocean, with currents labeled. Mark the highest surface height in the North Pacific Ocean. Highest is just east of Japan.

Mark the lowest surface height in the North Pacific Ocean. Lowest is in northwest Pacific, near the words ‘East Kamchatka Current” on the map.

Draw arrows showing the direction of the geostrophic currents between 10°N and 40°N based on this map and on where the sea surface is high and low. Arrows should follow exactly the contours that are shown (these are the surface currents), and should be from left to right in the region between 35 and 40°N (so following around the contours, will be from right to left between 10 and 20°N).

(d) Explain the direction of these currents that you have marked, relative to the high and low sea surface. You may refer to the equations above. Highest pressure is east of Japan and Taiwan (where the word “Kuroshio” appears on the map). The PGF points from the high to the low. Coriolis turns the water to the right, and so it goes clockwise around the large high pressure subtropical region and counterclockwise around the subpolar low pressure region.
12. (20 points)

(a) Heat flux at the ocean surface is the sum of **shortwave**, **longwave**, **latent** and **sensible** heat fluxes. Which *two* of these are radiative fluxes? **Shortwave and longwave**.

Of these two radiative fluxes, which one results in the ocean losing heat to the atmosphere? **Longwave (due to blackbody radiation)**.
(b) The plot above shows the four components of air-sea heat flux and the net flux (sum). They have been zonally averaged around the globe.

Between 20°N and 40°N, is the ocean gaining or losing heat? (circle one or write it out) losing heat

(c) Based on this plot, does the ocean most likely transport heat northward or southward between the equator and 35°N? Explain your answer. Northward heat transport, because the ocean is gaining heat in the tropics between about 10S and 20N, and losing heat poleward of this, so heat is transported northward to the heat loss region.

(d) On the map of surface heat flux, look at the Northern Hemisphere between 20°N and 40°N. Circle the areas of largest heat exchange. These are regions with labels -125 in the N. Pacific and -150 in the N. Atlantic. What ocean circulation features (currents) are located in these areas of the largest air-sea heat exchange? Kuroshio and Gulf Stream, respectively.

e) If the North Pacific heat transport at 24°N is 1 PW, and the volume transport of the Kuroshio is 100 Sv, calculate the difference in average temperature between the Kuroshio and the southward interior subtropical gyre flow. (Just a note: you can assume that the deep circulation here carries very little heat since the deep N. Pacific circulation is very weak compared with its upper ocean circulation.) Use the formula provided at top of exam, and approximate values provided there.
$H = \rho c_p V(T_o - T_i)$
Use $\rho = 1025 \text{ kg/m}^3$ and $c_p = 4000 \text{ J/kg°C}$.

$1 \text{ PW} = 1 \times 10^{15} \text{ W} = (1025 \text{ kg/m}^3)(4000 \text{ J/kg°C})(100 \times 10^6 \text{ m}^3/\text{sec}) (T_o - T_i)$

I am just asking for $(T_o - T_i) = \frac{(1 \times 10^{15} \text{ W})}{(1025 \text{ kg/m}^3)(4000 \text{ J/kg°C})(100 \times 10^6 \text{ m}^3/\text{sec})} = 2.44°C$

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13. (10 points)

(a) The plot shows measured (‘in situ’) temperature and potential temperature over the Mariana Trench in the Pacific (deepest place in the world ocean).
Label the curves:
- measured temperature $T$ right curve
- potential temperature $\theta$ left curve

(b) How do you know which one is potential temperature? (brief explanation)
(c) This plot shows potential density in the North Pacific, at 3 different locations.

Label an example of each of the following:
- Mixed Layer
- Pycnocline
- Pycnostad

Mixed layer is very thin, well mixed layer at sea surface
Pycnocline(s) are higher vertical gradient regions (where density changes fast, in these examples near 200 m (red and blue curves) and above and below the pycnostad on the third (black) curve.

Many of you answered that the abyssal water is a pycnostad. I decided not to give credit on this, because ‘pycnostad’ is used to refer to a low density layer embedded in the pycnocline.

14. (20 points)
The plot shows contours of potential density $\sigma_4$ relative to 4000 dbar.

(a) Relative to this reference pressure of 4000 dbar, which water parcel is denser, A or B?
Parcel A is denser (1044 kg/m$^3$).
Parcel B is about 43.1 kg/m$^3$

(b) Is colder water **more compressible** or **less compressible** than warmer water? (circle one)
(c) Suppose these two parcels, A and B, are brought to the sea surface adiabatically. What does it mean to move the water adiabatically? (What is conserved?)

Move them to sea surface without exchange of heat or salt, so both heat and salt are conserved.

- Some of you said that temperature is conserved, but it is actually heat that is conserved, so I took off a point.
- Many of you did not mention conservation of salt. So many that I decided not to take off a point, but you should understand that both heat and salt are conserved.

(d) At the sea surface, is their density difference (at the sea surface) likely to be GREATER or LESSER than it is at 4000 dbar? Explain your answer in terms of dependence of sea water compressibility on temperature.

Because the cold water is more compressible than the warm water, it will expand more as it goes up to the surface, so its density will decrease more rapidly than the warm water. This causes their densities to be more similar at the sea surface.

(e) On the plot, sketch the contours of potential density relative to 0 dbar. (This is schematic, but should include the proper orientation relative to the contours that are shown.) Explain the orientation of the contours based on your answer to (d).

Difficult to do in an answer key, but the contours will be rotated counterclockwise relative to the sigma 4 contours shown above, and so will slightly more vertical.

15. (20 points)

Water masses: On the attached potential temperature and salinity sections:

(a) Label the “4 layers” (upper, intermediate, deep, abyssal) See annotated figure.

(b) Label these two major water masses on the figure: See annotated figure.
- North Atlantic Deep Water (NADW)
- Antarctic Bottom Water (AABW)

(c) How did you locate NADW using these property sections? (short answer)
NADW is identified as a large-scale salinity maximum in the deep water layer.

(d) How did you locate AABW using these two property sections? (short answer)
AABW is identified as the cold bottom water beneath the NADW.

(e) How is AABW formed? How does this produce the property distribution that allowed you to identify AABW on the sections?
AABW is formed through brine rejection close to the coast of Antarctica (or just offshore or within the ice shelves, which are an extension of the continent). Therefore it is formed at freezing point temperature, which gives AABW its property of being very cold.